Pythagorean Theorem and its Converse

Mr. West, Geometry – Osborne High School:
[Trig – Day 1]

Name _____ **Key**
Block: _____ Date: _____

Recall: The Pythagorean Theorem is $a^2 + b^2 = c^2$

I. Use the Pythagorean Theorem to find the third side of each triangle
(Given: "a" and "b" are legs and "c" is the hypotenuse)

1. $a = 12, b = 16$

$12^2 + 16^2 = C^2$
$144 + 256 = 400$
$C^2 = 400 \quad C = \sqrt{400} = 20$

2. $a = 9, c = 41$

$9^2 + b^2 = 41^2$
$81 + b^2 = 1681$
$-81 \qquad -81$
$b^2 = 1600 \quad b = \sqrt{1600} \quad b = 40$

3. $a = \sqrt{13}, b = \sqrt{12}$

$(\sqrt{13})^2 + (\sqrt{12})^2 = C^2$
$13 + 12 = C^2$
$35 = C^2 \quad \sqrt{35} = C$

4. $b = 35, c = 37$

$a^2 + 35^2 = 37^2$
$a^2 + 1225 = 1369$
$a^2 = 144 \quad a = 12$

5. $a = 8, c = 12$

$8^2 + b^2 = 12^2$
$64 + b^2 = 144$
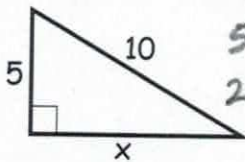$b^2 = 80 \quad b = \sqrt{80} \quad b = 4\sqrt{5}$

6. $a = 3\sqrt{2}, c = 6$

$(3\sqrt{2})^2 + b^2 = 6^2$
$18 + b^2 = 36 \quad b = \sqrt{18}$
$b^2 = 18 \quad b = 3\sqrt{2}$

7.

$5^2 + x^2 = 10^2$
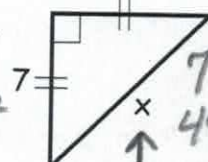$25 + x^2 = 100$
$x^2 = 75$
$x = \sqrt{75}$
$X = 5\sqrt{3}$

8.

$57^2 + 64^2 = x^2$
$3249 + 4096 = x^2$
$7345 = x^2$
$\sqrt{7345} = x$

9.

$7^2 + 7^2 = x^2$
$49 + 49 = x^2$
$98 = x^2$
$\sqrt{98} = x$
$7\sqrt{2} = x$

10.

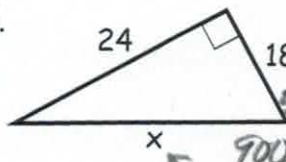$x^2 + 8^2 = 10^2$
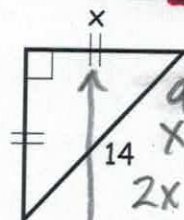$x^2 + 64 = 100$
$\sqrt{x^2} = \sqrt{36}$
$x = 6$

11.

$24^2 + 18^2 = x^2$
$576 + 324 = x^2$
$900 = x^2$
$30 = x$

12.

$a^2 + b^2 = 14^2$
$x^2 + x^2 = 14^2$
$2x^2 = 14^2$
$\dfrac{2x^2}{2} = \dfrac{196}{2}$
$x^2 = 98$
$x = 7\sqrt{2}$

II. (The two shorter sides must add up (sum) to more than the longer side.)
Which of the following numbers can be the lengths of sides of a triangle? If the lengths can make a triangle, which type? SHOW YOUR WORK to justify the answer.

1. 9, 12, 16 ✓
$9 + 12 = 21$
$21 > 16$ ✓
$9^2 + 12^2 = 225$
$16^2 = 256$
$256 > 225$
Obtuse

2. 12, 13, 7 ✓
$7 + 12 = 19$
$19 > 13$ ✓
$7^2 + 12^2 = 193$
$19^2 = 361$
$361 > 193$
Obtuse

3. 6, 8, 10
$6 + 8 = 14$
$14 > 10$ ✓
$6^2 + 8^2 = 100$
$10^2 = 100$
rt. △

4. 34, 21, 24
$21 + 24 = 45$
$45 > 34$ ✓
$21^2 + 24^2 = 1017$
$34^2 = 1,156$
Obtuse

5. 11, 7, 18
$11 + 7 = 18$
$18 = 18$
Not a △ !

6. 5, 8, 7
$5 + 7 = 12$
$12 > 8$ ✓
$5^2 + 7^2 = 74$
$8^2 = 64$
Acute

7. 12, 15, 9
$12 + 9 = 21$
$15 < 21$
Not a △ !

8. 3, 15, 17
$3 + 15 = 18$
$18 > 17$ ✓
$3^2 + 15^2 = 234$
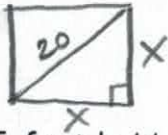$17^2 = 289$
Obtuse

9. 14, 21, 6
$6 + 14 = 20$
$20 < 21$
Not a △ !

III. Use the **Pythagorean Theorem** to solve these problems. $a^2 + b^2 = c^2$
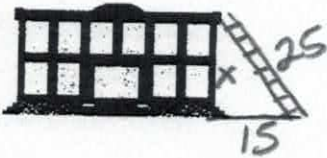Draw pictures to illustrate the problem.

1. A square has a diagonal with a length of 20 cm. What is the measure of each side?

45, 45, 90 triangle

$2x^2 = \dfrac{400}{2}$ $\sqrt{x^2} = \sqrt{200}$

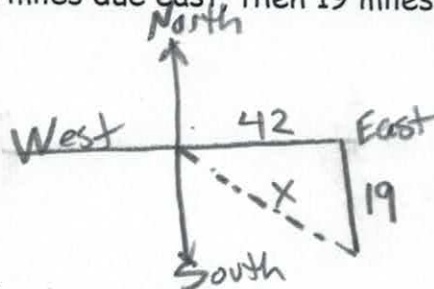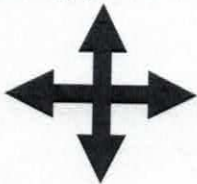$x^2 + x^2 = 20^2$ $\dfrac{400}{2}$ $x = 10\sqrt{2}$

2. A 25-foot ladder is leaning against a building. The foot of the ladder is 15 feet from the base of the building. How high is the top of the ladder along the building?

$x^2 + 15^2 = 25^2$ $\sqrt{x^2} = \sqrt{400}$

$x^2 + 225 = 625$ $x = 20$

$x^2 = 400$
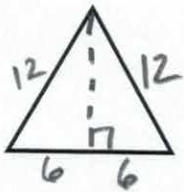
Bldg is 20 ft. tall!

3. Ashley travels 42 miles due east, then 19 miles due south. How far is Ashley from the starting point?

North   West   42   East   19   South

$42^2 + 19^2 = x^2$

$1764 + 361 = x^2$

$2125 = x^2$

$\sqrt{2125} = x$ $5\sqrt{85} = x$

≈ 46.09 miles

4. What is the length of the altitude of an equilateral triangle if a side is 12 cm?

$6^2 + x^2 = 12^2$ $\sqrt{x^2} = \sqrt{108}$

$36 + x^2 = 144$ $x = 6\sqrt{3}$

$x^2 = 108$ ≈ 10.39 ft.

5. A pole was 15 feet tall before a storm broke it. If there is still 3 feet of the pole standing upright and the rest is toppled over at an angle, how far is the "top" of the pole from the base?
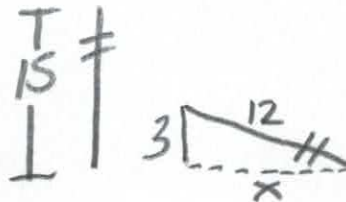
$x^2 + 3^2 = 12^2$ $\sqrt{x^2} = \sqrt{135}$

$x^2 + 9 = 144$ $x = 3\sqrt{15}$

$x^2 = 135$ ≈ 11.62 ft.

6. Sam, who is 6 feet tall, is flying a kite. He has let out 1125 meters of string which he stakes into the ground. The stake is 75 meters from a tree which the kite is directly above. How high is the kite?

✳ Who cares how tall Sam is? It doesn't matter since he stakes the string to the ground.
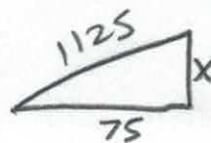
$x^2 + 75^2 = 1125^2$

$x^2 + 5625 = 1,265,625$

$x^2 = 1,260,000$

≈ 1122.5 ft. $x = 300\sqrt{14}$